International Journal of Communication Networks and Information Security (IJCNIS) Vol. 1, No. 3, December 2009

# Analysis and Design of Printed Fractal Antennas by Using an Adequate Electrical Model

Ferchichi Abdelhak[1], Fadlallah Najib[2], Sboui Noureddine[1], Gharssallah Ali[1]

1-Unit of research Circuits and Electronics Systems High Frequency
Faculty of Science, University El Manar
Tunis, Tunisia
2- Radiocom Team, Lebanese University
IUT Saida, Lebanon
abdelhakferchichi@yahoo.fr, n_fadlallah@yahoo.com

***Abstract:*** In this paper, we propose two electrical models of carpet and gasket Sierpenski patch antenna. In our approach, we replace the antenna structure by its equivalent rectangular patch. Then, we build the electrical model in which we extract the resonant circuit RLC parameters. Return losses of the models are compared to physical patch for frequency band [1GHz, 3GHz]. Simulation results given by our approach, show that the antenna and its model have the same resonant frequency at 2.45GHz but with a little difference in Bandwidth.

***Keywords:*** Fractal antenna, Sierpenski carpet, Sierpenski Gasket, Radio Frequency Identification RFID, electrical model

## 1. Introduction

The booming progress of wireless communication systems and the variety applications increase the demand of microstrip patch antenna. They are used in many applications because of their low-profile, ease of fabrication, small size and low cost. One of the solutions is the fractal antenna. It is made in order to obtain a small and multiband patch. The sierpenski antenna is the first example of a multiband fractal antenna and it gives a small patch [1]. Those types of antennas were introduced in different applications like in RFID application which consist one of the new techniques of automatic identification and where the system size has a great importance and it depends essentially on the antenna size [2]. In this way, we design a patch based on sierpenski carpet and sierpenski gasket antenna by using ADS simulator. In order to analyze those structures, we proposed the equivalent electrical model of Sierpenski and we compared it to a physical patch. The two models are based on the electrical model of square patch.

This paper is organized as follows. In section 2, a proposed equivalent circuit of the first iteration of Sierpenski carpet patch antenna is presented. In section 3, the electrical model of the first iteration of sierpenski gasket patch is developed. Finally, section 4 deals with the different conclusions.

## 2. Sierpenski carpet antenna

### 2.1. Design of square antenna

Figure (1) shows the geometry of a square microstrip patch on a dielectric substrate with a ground plane. The antenna have an edge a = 36mm and it is mounted on a substrate material with a thickness h=3.2mm, a dielectric constant $\varepsilon_r = 2.6$ and loss tangent (tangδ) = 0.002. The dimension 'a' of the square edge is calculated using equation (1):

$$a = \frac{c}{2 f_r \sqrt{\varepsilon_r}} \quad (1)$$

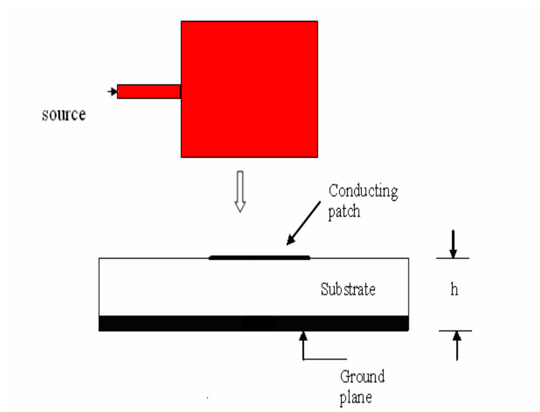


**Figure 1**. Square patch

### 2.2. The electrical model of rectangular patch antenna

The square patch electrical parameters can be calculated from the input impedance of a rectangular patch excited by a transmission line which is given in [3] by equation 2:

$$Z_{in} = R + jX \quad (2)$$

$$Z_{in}(f) = \frac{R}{1+Q_T^2\left[\frac{f}{f_r}-\frac{f_r}{f}\right]^2} + j\left[\frac{RQ_T\left[\frac{f}{f_r}-\frac{f_r}{f}\right]}{1+Q_T^2\left[\frac{f}{f_r}-\frac{f_r}{f}\right]^2}\right]$$

R is the resonant resistance; QT is the Quality factor, f is the operating frequency and $f_r$ is the resonant frequency.

To calculate the parameters of the rectangular electrical model we used the formula below.

#### *2.2.1. Resonant resistance R*

The resonant resistance is calculated using equation (3) as in [3]:

$$R = \frac{Q_T\ H}{\pi f_r\ \varepsilon_{dyn}\varepsilon_0\ A}\cos^2\left(\frac{\pi\ x_0}{a}\right) \quad (3)$$

$f_r$ : resonant frequency
$Q_T$ : Quality factor
$\varepsilon_{dyn}$ : dynamic permittivity
$x_0$ : the distance of the feed point from the edge of the patch.
a : length of square
H : thickness of dielectric
A : aire of square

$$Q_T = \left(\frac{1}{Q_R} + \frac{1}{Q_C} + \frac{1}{Q_D}\right)^{-1} \quad (4)$$

$$Q_R = \frac{c_0\sqrt{\varepsilon_{dyn}}}{4 f_r\ H} \text{ : Radiation quality factor} \quad (5)$$

$$Q_D = \frac{1}{Tg\delta} \text{: losses in the dielectric} \quad (6)$$

$$Q_C = \frac{0.786\sqrt{f_r Z_{a0}(W)H}}{P_a} \text{:Losses in the conductor} \quad (7)$$

$$Z_a(W) = \frac{60\pi}{\sqrt{\varepsilon_r}}\left(\frac{W}{2H} + 0.441 + 0.082\left(\frac{\varepsilon_r - 1}{\varepsilon_r^2}\right) + \frac{(\varepsilon_r + 1)}{2\pi\varepsilon_r}\left(1.451 + Ln\left(\frac{W}{2H} + 0.94\right)\right)\right) \quad (8)$$

is the impedance of an air filled microstrip line

$$Z_{a0}(W) = Z_a(W, \varepsilon_r = 1)$$

$$P_a = \frac{2\pi\left(\frac{W}{H} + \frac{W/(\pi H)}{W/2H + 0.94}\right)\left(1 + \frac{H}{W}\right)}{\left(\frac{W}{H} + \frac{2}{\pi}Ln\left(2\pi\exp\left(\frac{W}{2H} + 0.94\right)\right)\right)^2}, \quad W/H > 2 \quad (9)$$

$$\varepsilon_{dyn} = \frac{C_{dyn}(\varepsilon)}{C_{dyn}(\varepsilon_0)} \quad (10)$$

$$C_{dyn}(\varepsilon) = \frac{\varepsilon_0\varepsilon_r A}{H\,\gamma_n\gamma_m} + \frac{1}{2\gamma_n}\left(\frac{\varepsilon_{reff}(\varepsilon_r, H, W)}{c_0 Z(\varepsilon_r = 1, H, W)} - \frac{\varepsilon_0\varepsilon_r A}{H}\right) \quad (11)$$

$$\gamma_j = \begin{cases} 1, & j = 0 \\ 2, & j \neq 0 \end{cases}$$

$$Z(w, H, \varepsilon_r = 1) = \frac{377}{2\pi}Ln\left(\frac{f(W/H)}{w/h} + \sqrt{1 + \left(\frac{2}{W/H}\right)^2}\right) \quad (12)$$

$$f(W/H) = 6 + (2\pi - 6)\exp\left(-\left(\frac{30.666}{W/H}\right)^{0.7528}\right) \quad (13)$$

### *2.2.2. Capacitance C, Inductance L*

To determine the capacitance C we use the formula of $C_{dyn}$.

To determine L, we know that:

$$\omega_{res} = 2\pi f_r \quad (14)$$

$$\omega_{res} = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_{res}^2 C} \quad (15)$$

### 2.3. Fractal antenna

The term fractal means broken (fragmented), it was defined by the mathematician Benoit Mandelbrot.The main geometric characteristic of fractal antenna are:

- Infinite perimeter
- Finite surface
- Fractal dimension : D

The general concept of fractal can be applied to develop various antenna elements. Applying fractals to antenna elements allows for smaller and resonant antennas that are multiband frequency. Furthermore, the dimension of geometries can be defined through Euclidean dimension and self-similarity dimension [4].

### 2.4. Geometry of Sierpenski carpet antenna:

The Sierpenski carpet antenna is designed from an initial square patch. The first iteration is constructed by devised the square into nine small square and removed the centre one. The same procedure is repeated from iteration to other [5]. This kind of fractal antenna is used to develop multiband antenna, those antenna became very useful in new telecom system as RFID system.

Figure (5) shows three iterations of Sierpenski carpet antenna which is mounted on substrate material with a thickness h=3.2mm, a dielectric constant $\varepsilon_r = 2.6$ and loss tangent (tangδ) = 0.0002. The patch is exited by a transmission line and the dimension is determined to resonate at 2.45GHz.

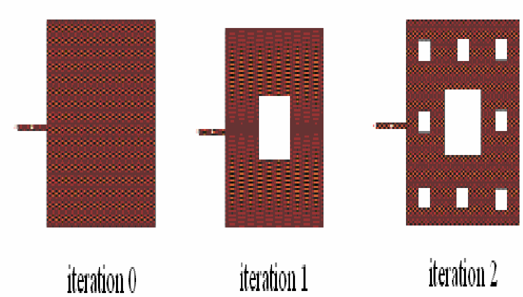


**Figure 2**. Three first iteration sierpenski carpet antenna

### 2.5. A proposed electrical model to Sierpenski carpet antenna

In order to modelizate the first iteration of sierpenski carpet antenna, we concentrate our work to search its equivalent square patch. After many test, we demonstrate that the first iteration have a similar characteristic of a square patch with a great size which explain the use of this antenna to reduce the antenna size. Thus, the structure can be modelizated by an RLC resonant circuit. The model parameters are calculated using the formulas developed in section 1.

The model is simulated and then compared to the physical patch. We can see that the two results are similar. The structure and its model have the same resonant frequency but with a little difference in Band width because of the difference between calculated and simulated losses figure (4). The proposed present a resonant frequency equal to 2.45GHz with a band width about 100MHz, a gain and directivity are equal to 5.41dB and 6.21dB respectively.

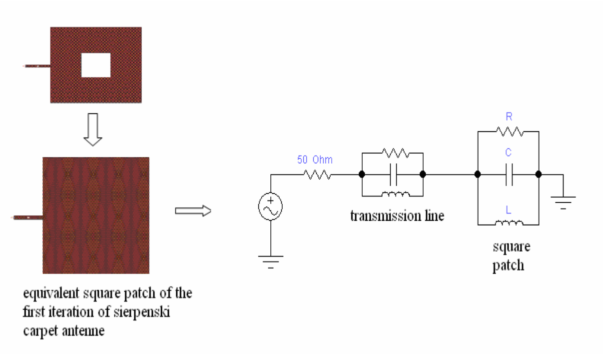


**Figure 3**. Electrical model of the first iteration of sierpenski carpet patch

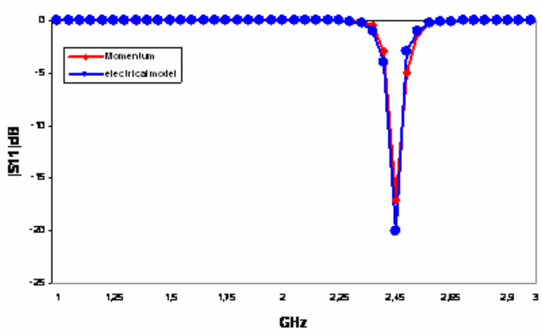


**Figure 4.** Return loss of physical, first iteration of sierpenski carpet antenna with C=0.119 nF, L= 35.46 pH, R= 55.51 Ohm

Our proposed approach can be generalised to modulate the three iterations. We applied an iterative processes to determine the equivalent square to the sierpenski carpet iteration: "from the iteration i we determine the equivalent structure in iteration (i-1) and the same procedure is repeated until having the equivalent square patch to the desired iteration of sierpenski carpet antenna" figure (5).

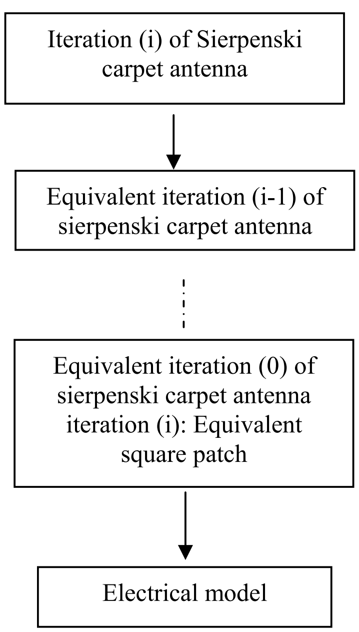


**Figure 5.** Process of electrical model to Sierpenski Carpet iteration 'i'

The proposed diagram is valid just for the three first iteration of sierpenski carpet patch because from the fourth iteration the proprieties antenna risks to be destroyed.

## 3. Sierpenski Gasket Antenna:

### A model of triangular patch antenna

(Design of the Triangular patch antenna)

Figure (6) shows the geometry of an equilateral triangular microstrip patch on a dielectric substrate with a ground plane. The antenna is mounted on a substrate material with a thickness h=3.2mm, a dielectric constant $\varepsilon_r = 2.6$ and loss tangent (tangδ) = 0.0002. The dimension of this triangle is calculated using equation (16):

$$f_r = \frac{2c}{3a\sqrt{\varepsilon_r}} \sqrt{m^2 + n^2 + mn} \quad (16)$$

For the fundamental mode, we have m=1, n= 0, a = 50 mm is the length of the triangle, 'c' is the velocity of light and $\varepsilon_r$ is the dielectric constant.

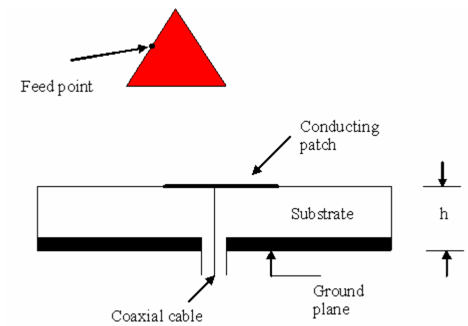


**Figure 6**. *Triangular patch*

The patch resonates at 2.45GHz (one of the frequencies used in RFID application). It represents acceptable parameters in RFID systems with a gain about 6.41dB and directivity equals to 7.81dB.

### A proposed electrical model

The recent study work of microstrip patch have demonstrate that the triangular patch have a radiation characteristic similar to a rectangular patch but with a reduced dimensions [6]. For this reason, to studies our antenna, we can replace the triangular electrical model by its equivalent electrical rectangular model. For that reason the triangular patch can be modulated by an RLC resonant circuit. The impedance Z of the patch is calculated used the formulas propose in section 1 where we introduce the reactance due to the coaxial probe Figure (7). The inductive reactance of Coax is calculated via the equation below [3]:

$$X_L = \frac{377\, fH}{c_0} Ln\left(\frac{c_0}{\pi f d_0 \sqrt{\varepsilon_0}}\right) \quad (17)$$

$d_0$ : is the diameter of the probe

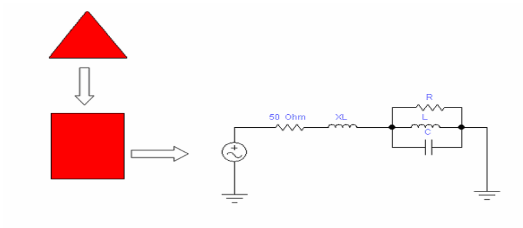


**Figure 7**. Electrical model

The model is simulated and then compared to physical patch. The results are represented in figure (8). The triangular patch and its electrical model present a resonant frequency about 2.45GHz but with a difference in Band width because of the difference between calculated and simulated losses. The gain and directivity equal respectively 7.41dB and 7.81dB.

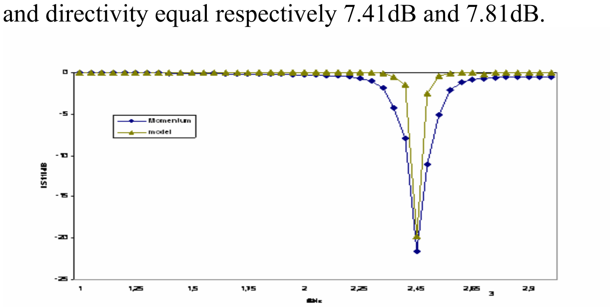


**Figure 8.** Return loss for both physical and model patch with C=0.168 nF**,** L= 25.11 pH, R= 66.66 Ohm, $X_L = 1nH$ (triangular patch).

**A Sierpenski gasket antenna**

**Geometry of Sierpenski gasket antenna**

The Sierpenski gasket is constructed by subtracting a central inverted triangle from main triangle. After the subtraction, three equal triangles stay on the structure. Then the same procedure is repeated to the remaining triangles [7]. The number of triangles Nn, length of a side of triangle Ln and the fractional area An can be calculated after every iteration [8]:

$$N_n = 3^n \quad (18)$$

$$L_n = \frac{1}{2^n} \quad (19)$$

$$A_n = L_n^2 N_n = \left(\frac{3}{4}\right)^n \quad (20)$$

Figure (9) shows three iterations of Sierpenski gasket antenna which is mounted on substrate material with a thickness h=3.2mm, a dielectric constant $\varepsilon_r = 2.6$ and loss tangent (tangδ) = 0.002. The patch is exited by a coaxial feed line at the middle of the right side.

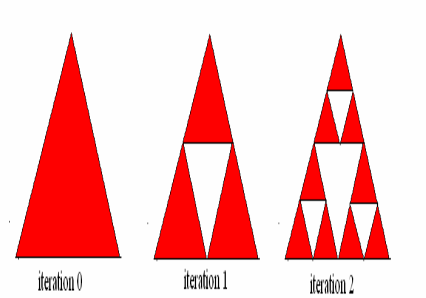


**Figure9**. Three first iteration Sierpenski gasket antennas

**Electrical model of Sierpenski Gasket antenna**

In order to modelizate sierpenski gasket antenna, we apply the model of triangular patch to every elementary triangle. For the fist iteration, we replace the three elementary triangles by its electrical model in order to obtain the whole electrical model of Sierpenski gasket. Finally, we calculate the parameters model using the formula developed in section 1 to determine the impedance Zi to each elementary triangle with R=51Ω, C = 0.1nF and L= 0.1nH. The electrical model is represented in figure (10).

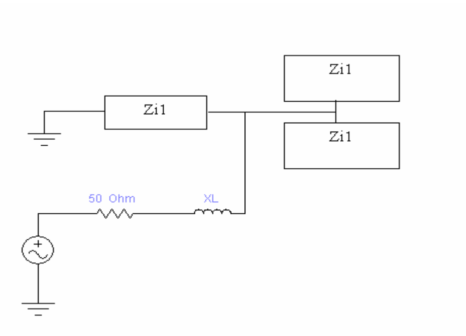


**Figure 10**. First electrical model of the first iteration of sierpenski gasket patch

In order to validate our model, we compare the result to a second model. The second electrical model is proposed by Walter Arrighetti, Peter De Cupis and Giorgio Gerosa in [9]. In their model, they assume that every triangle is composed by dissipative elements and the interconnections are LC-parallel resonating on the resonant frequency figure (11). They use a resistance about a micro- ohm, as for the capacitance is determined using the formula of parallel plate capacitor. And then the inductance L is determined by using the formula given in equation.

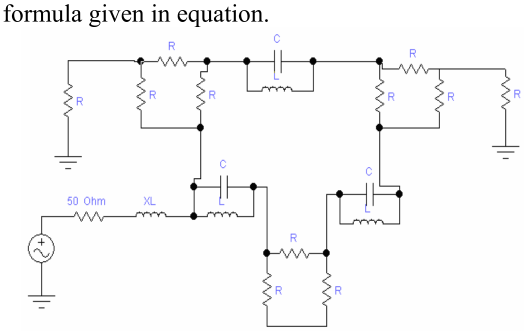


**Figure 11**. Second electrical model of the first iteration of sierpenski fractal patch

The two models are simulated and then compared to physical patch. The results are represented in figure (12). We can see that the two results are similar. The antenna and the electrical model have the same resonant frequency but with a little difference in Band width because of the difference between calculated and simulated losses. The proposed present a resonant frequency equal to 2.45GHz with a band width about 80MHz, a gain and directivity equal respectively 4.65dB and 5.62dB.

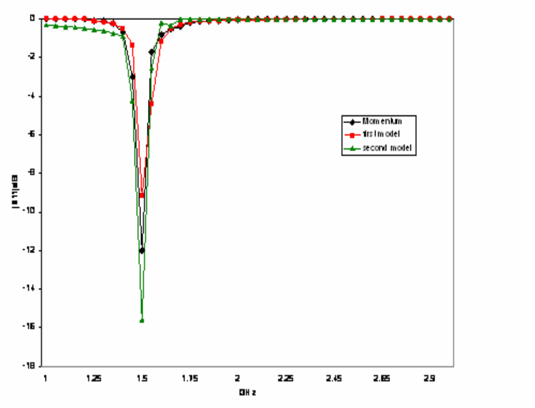


**Figure 12.** Return loss of physical, first and second model sierpenski antenna

## 4. Conclusion

Two easy electrical model of sierpenski gasket and sierpenski carpet patch are proposed. The models are determined by equivalent patch of the fractal antenna. The use of electrical model represents good results in comparison to experimental results in the first iteration. The importance of this approach is the possibility to analyze and calculate the antenna parameters early before conception using numerical formulas.

## References

[1] John P. Gianviffwb and Yahya Rahmat-Samii “Fractal Antenna : A Novel Antenna Miniaturization Technique, and Applications”, IEEE Transactions on Antennas and Propagation, Vol 44, NO 1, February 2002

[2] Klaus Finkenzeller “RFID HANDBOOK”, Second Edition, 2003

[3] Settapong Malisuwan, Monton Charoenwattanaporn, Ut Goenchanart “ microstrip antenna Antenna for wireless lan by Appling modified smith-chartrepresentation”, International journal of the computer, the Internet and management, Vol 11, pp 34-44, 2003

[4] M.K.A. Rahim, A.S. Jaafar, and M.Z.A. Abdul Aziz “Sierpenski Gasket Monopole Antenna Design”, Asia Pacific Conference on Applied Electromagnetic Proceedings, 2005

[5] Ban-leong Ooi “Modified Contour Integral Analysis for Sierpinski Fractal Carpet Antennas With and Without Electromagnetic Band Gap Ground Plane”, IEEE Transactions on Antennas and Propagation, Vol 52, NO 5, May 2004

[6] Nasimuddin and A. K. Verma Amir “Fast and accurate model for analysis of equilateral triangular patch antenna”, Journal of Microwaves and Optoelectronics, Vol 3, pp 99-110, April 2004

[7] J. Parron, J. Romeu, J.M. Rius, J.R Mosig “Method of Moments Enhancement Technique for the Analysis of Sierpinski Pre-Fractal Antennas”, IEEE Transactions on Antennas and Propagation, Vol 51, NO 8, August 2003

[8] Douglas H. Werner, Randy L. Haupt, and Pingjuan L. WerneJ, Fractal Antenna Engineering: The Theory and Design of Fractal Antenna Arrays, IEEE Antenna and Propagation Magazine, pp37-57, 1999

[9] Walter Arrighetti, Peter De Cupis and Giorgio Gerosa “Circuit model for Sierpenski gasket antennas”, arXiv, physics, octobre2005