

Chaotic Maps: A Tool to Enhance the Performance of OFDM Systems

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Abstract: This paper presents a chaotic interleaving approach for efficient data transmission with orthogonal frequency division multiplexing (OFDM) over fading channels. The binary data is interleaved with the proposed approach prior to the modulation step. The chaotic Baker map is used in the proposed interleaving approach. In addition to reducing the channel effects on the transmitted data, the proposed chaotic interleaving approach adds a degree of encryption to the transmitted data. The performance of the proposed approach is tested by the transmission of images over Rayleigh fading channels with chaotic interleaving. Experimental results show that images are received with higher peak signal to noise ratios (PSNRs) if chaotic interleaving is applied.

Keywords: Adaptive modulation, orthogonal frequency division multiplexing (OFDM), channel coding, bit error rate (BER), throughput.

1. Introduction

OFDM is a multicarrier modulation technology which has efficient spectrum utilization to support the transmission of high data rates [1,2]. Transmission of images and multimedia with the OFDM technology has attracted the attention of several researchers [3-7]. Furthermore, several coding techniques have been investigated for efficient transmission of images with OFDM over wireless channels [3,4]. Despite the efficiency of these coding techniques, they add much redundancy to the transmitted data leading to lower channel utilization.

Another investigated approach for efficient transmission of images is the adaptive OFDM [5]. In this approach, the subcarrier mapping is performed according to the channel conditions by using a channel estimation algorithm. Although this approach takes the channel conditions into consideration, it is difficult to sense the channel conditions at the transmitter and to perform channel estimation before data transmission.

To avoid the complexity of the channel estimation at the transmitter, Salah et al. have proposed an adaptive OFDM scheme which does not require channel estimation at the transmitter [6]. This scheme is based on using an unequal cyclic time guard (UTG) with OFDM. This scheme has succeeded in achieving a good performance at high signal to noise ratios (SNRs). At low SNRs, the need for coding was unavoidable. The failure of this scheme to operate at low SNRs without strong coding techniques has limited its applicability.

The evolution of multiple input multiple output (MIMO) and beamforming techniques in wireless communications has opened a new field of research for efficient image

transmission over wireless channels [7,8]. MIMO techniques can improve the efficiency of image transmission through the addition of diversity to the system. Beamforming can be used for interference reduction. These new technologies can achieve a high efficiency in image transmission systems but with increased complexity and cost.

Another approach which will be considered in this paper is the performance enhancement of image transmission systems through data interleaving. It is known that image transmission over wireless channels may face severe adverse conditions, especially burst errors where errors are likely to occur in clusters. To reduce the effect of these burst errors, binary data interleaving of the images prior to modulation is required [9-11]. There are some simple interleaving techniques such as block interleaving [12,13]. The performance of such simple interleavers is limited. Therefore, there is a need for more powerful interleavers.

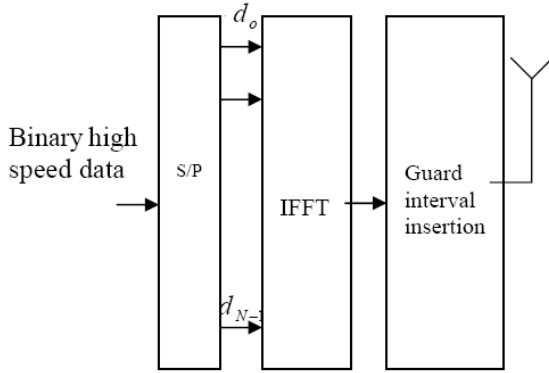
Chaotic maps can be used to build strong interleavers. Thus, in this paper, a chaotic interleaver scheme based on chaotic Baker map is introduced. The proposed interleaver is applied to the image in binary format prior to modulation. Due to the strong interleaving ability of chaotic maps, the proposed approach can combat the channel effects without a need for complicated coding schemes for error detection and correction or adaptation in the modulation scheme. Another advantage of the proposed chaotic interleaving scheme is that it achieves a degree of encryption in the transmitted data which adds more security to the image transmission process.

This paper is organized as follows. Section 2 provides an overview of the OFDM system model used for image transmission. Section 3 discusses the channel characteristics in OFDM systems. Section 4 presents the fundamentals of the chaotic Baker map. Section 5 explains the proposed OFDM system model with chaotic interleaving. The simulation results are presented in Section 6. Finally, Section 7 gives the concluding remarks.

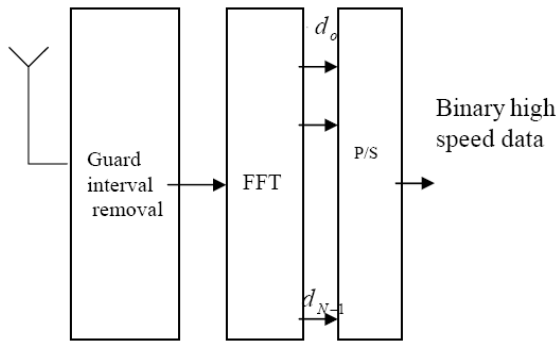
2. OFDM SYSTEM MODEL

OFDM is one of the most powerful communication technologies which are implemented in current wireless communications systems. In OFDM, the transmitted high-speed data is converted into parallel data of N subchannels [1,2]. Then, the transmitted data of each parallel subchannel is modulated by an adequate modulation scheme like quadrature phase shift keying (QPSK). These modulated data are fed into an inverse fast Fourier transform (IFFT) to generate the multicarrier OFDM signal. In order to avoid the intersymbol interference (ISI) which occurs in multipath

channels, guard intervals are inserted between frames [1,2]. The block diagram of the traditional OFDM system is shown in Figure (1).



(a) Transmitter.



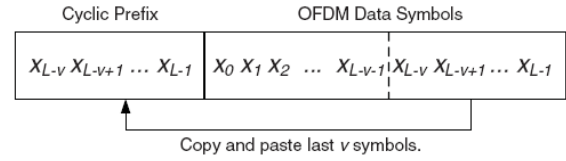
(b) Receiver.

Figure 1. OFDM System Model.

Guard intervals are either in the form of added zeros or a cyclic prefix (CP). The addition of a CP is the most widely used approach. The key for making OFDM realizable in practice is the use of the fast Fourier transform (FFT) and the inverse fast Fourier transform (IFFT), which have low complexity. In order for the IFFT/FFT to create an ISI-free channel, the channel must appear to provide a circular convolution [1,2]. Adding a CP to the transmitted signal, as shown in Figure (2), enables the circular convolution property of the channel.

If the maximum channel spread delay has a duration of $v+1$ samples, adding a guard band of at least v samples between OFDM symbols makes each OFDM symbol independent of those coming before and after it, and so only a single OFDM symbol can be considered. If we assume that the symbol time is T_s , and the CP time is T_g , the total symbol duration will be [1,2]:

$$T_{total} = T_g + T_s. \quad (1)$$

**Figure 2.** Addition of the CP to an OFDM symbol.

When the guard interval is longer than the channel impulse response or the multipath delay, the effect of the ISI can be eliminated. However, the intercarrier interference (ICI) or the in-band fading still exists. The ratio between the guard interval and the usual symbol duration is application dependent, because the insertion of a guard interval will reduce the data throughput. After the insertion of the guard interval, the transmitted OFDM signal is given by [14]:

$$s'(t) = \sum_{k=-\infty}^{\infty} \sum_{i=0}^{N-1} d(k) \exp(j2\pi f_i(t - kT_{total})) \times f'(t - kT_{total}) \quad (2)$$

where $d(k)$ is the data sequence and $f'(t)$ is the modified pulse waveform of each symbol which is defined as [14]:

$$f'(t) = \begin{cases} 1 & (-T_g \leq t \leq T_s) \\ 0 & (t < -T_g, t > T_s) \end{cases} \quad (3)$$

The OFDM signal is then transmitted through the channel. The transmitted data $s'(t)$ is then affected by a multipath fading channel and contaminated by additive white Gaussian noise (AWGN). The received signal is defined as [14]:

$$r(t) = \int_0^{\infty} h(\tau) s'(t - \tau) d\tau + n(t) \quad (4)$$

where $h(\tau)$ is the impulse response of the radio channel at time τ , and $n(t)$ is the AWGN.

The received signal $r(t)$ is filtered by a bandpass filter, which is assumed to have a sufficiently wide passband to avoid distorting the signal. The guard interval is then removed and an orthogonal detector is applied to the signal in order to obtain the Fourier coefficients of the signal in the observation periods. The output $\hat{d}_i(k)$ of the FFT circuit of the i^{th} OFDM subchannel is given by [14]:

$$\hat{d}_i(k) = 1/T_s \int_{kT_{total}}^{T_s + kT_{total}} r(t) \exp(-j2\pi f_i(t - kT_{total})) dt \quad (5)$$

3. Rayleigh Fading Channel

The channel in OFDM is characterized by various obstacles and reflections which have a large influence on the signals during the propagation of radio waves from the base station to the mobile station. The radio waves transmitted from a base station radiate in all directions, and hence the receiver receives many reflected waves from various obstacles, diffracted waves, scattered waves and the direct waves from the base station. Since the path lengths of the direct, reflected, diffracted, and scattered waves are different, the

travel time for each wave in order to reach the mobile station will be different as well. Furthermore, the phases of the incoming waves vary because of reflections.

As a result, the receiver receives a superposition of several waves having different phases and arrival times. The reception environment that is characterized by a superposition of delayed waves is called a multipath propagation environment. In a multipath propagation environment, the received signal is sometimes intensified or weakened. This phenomenon is called multipath fading and the signal level of the received wave changes from moment to moment. The multipath fading raises the error rate of the received data [14].

Let us begin with the mechanism by which fading occurs. The delayed wave with incident angle θ_n is given by [14]:

$$r_n(t) = \text{Re}[e_n(t) \exp j(2\pi f_c t)] \quad (6)$$

where $\text{Re}[\]$ indicates the real part of a complex number that gives the complex envelope of the incoming wave from the n^{th} directions of a multipath channel. $e_n(t)$ is generated from the propagation path with length L_n between the base station and the mobile station and it is given by [14]:

$$e_n(t) = R_n(t) \exp j \left(-\frac{2\pi(L_n - vt \cos \theta_n)}{\lambda} + \phi_n \right) \quad (7)$$

R_n and Φ_n are the envelope and the phase of the n^{th} incoming wave, respectively, v is the speed of mobile station (m/s), and λ is the wavelength.

The n^{th} incoming wave has a shift in the carrier frequency by the Doppler shift f_d (Hz), which is described as [14]:

$$f_d = (v/\lambda) \cos(\theta_n) \quad (8)$$

It has a maximum value of v/λ , when the incoming wave comes from the running direction of the mobile station at $\cos(\theta_n) = 1$ [14]. It is noted that, the envelope fluctuations follow a Rayleigh distribution, while the phase fluctuations follow a uniform distribution. These Rayleigh distribution fluctuations of the envelope deteriorate the received signals, severely. So, there is a need for a strong interleaving mechanism to reduce the effect of fading on the received signals.

4. CHAOTIC INTERLEAVING

The Baker map is a chaotic map that generates a permuted version of a square matrix. In its discretized form, the Baker map is an efficient tool to randomize a square matrix of data. The discretized map can be represented for an $M \times M$ matrix as follows [15-20]:

$$B(r, s) = \left[\frac{M}{n_i} (r - M_i) + s \bmod \left(\frac{M}{n_i} \right), \frac{n_i}{M} \left(s - s \bmod \left(\frac{M}{n_i} \right) \right) + M_i \right] \quad (9)$$

where $B(r, s)$ are the new indices of the data item at (r, s) , $M_i \leq r < M_i + n_i$, $0 < s < M$ and $M_i = n_1 + n_2 + \dots + n_i$.

In steps, the chaotic interleaving is performed as follows:

1. An $M \times M$ square matrix is divided into k vertical rectangles of height M and width n_i .
2. These vertical rectangles are stretched in the horizontal direction and contracted vertically to obtain an $n_i \times M$ horizontal rectangle.
3. These rectangles are stacked as shown in Figure (3-a), where the left one is put at the bottom and the right one at the top.
4. Each $n_i \times M$ vertical rectangle is divided into n_i boxes of dimensions $M/n_i \times n_i$ containing exactly M points.
5. Each of these boxes is mapped column by column into a row as shown in Figure (3-b).

Figure 3 shows an example of chaotic interleaving of an (8×8) square matrix. The secret key, $S_{\text{key}} = (n_1, n_2, n_3) = (2, 4, 2)$.

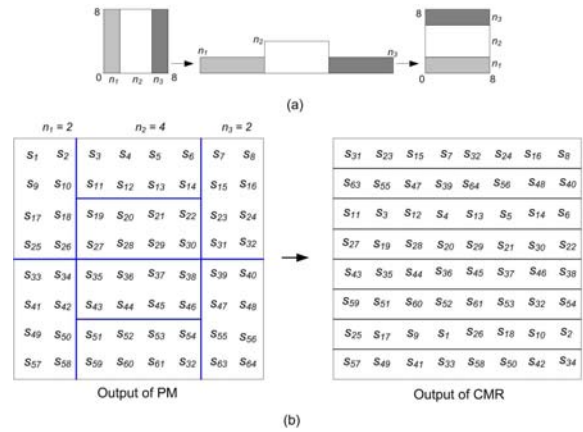


Figure 3. Chaotic interleaving. (a) Discretized Baker map. (b) Randomization of a 8×8 block.

5. THE PROPOSED OFDM SYSTEM MODEL

The proposed image transmission OFDM system model in this paper consists mainly of 3 stages which are an image data formatting stage, a chaotic interleaving stage which is performed on the binary image data, and an OFDM modulation stage. The block diagram of the proposed system model is shown in Figure (4).

At the transmitter, the chaotic interleaving is applied to the image in its binary format, and hence the image binary data is randomized before the OFDM modulator step. The steps of the proposed OFDM system model can be summarized as follows:

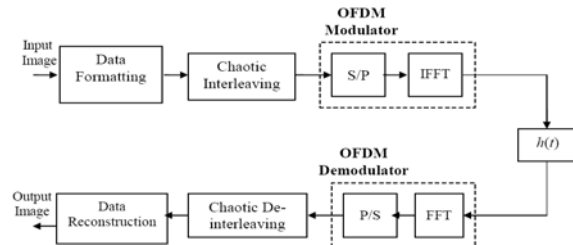


Figure 4. Block diagram of the proposed OFDM system model.

1. The image is transformed to the binary format, pixel by pixel, in matrix form.
2. The non-square binary matrix is reshaped to an $M \times M$ square structure.
3. Chaotic randomization is applied to this square matrix.
4. After the randomization process, the square binary matrix is reshaped again to its original dimensions.
5. An OFDM modulation step is performed on the binary data.
6. At the receiver, after the OFDM demodulation step, the received binary data is reshaped again to a square matrix and chaotic derandomization is applied to this matrix
7. The square binary matrix after derandomization is reshaped again into its original dimensions.
8. The image is retrieved from the binary data.

6. SIMULATION RESULTS

In this section, simulation experiments are performed to test and evaluate the proposed chaotic interleaving scheme and to compare it with normal OFDM. The PSNR is used to measure the quality of the reconstructed images at the receiver. It is defined as follows:

$$PSNR = 10 \log_{10} \left(\frac{f_{\max}^2}{MSE} \right) \quad (10)$$

where f_{\max} is the maximum possible pixel value of an image f . For 8 bit pixels, $f_{\max} = 255$. The MSE refers to the mean square error between the original image and the received image. For an $N \times N$ monochrome image, the MSE is defined as follows:

$$MSE = \frac{\sum_{i=1}^N \sum_{j=1}^N [f(i, j) - \hat{f}(i, j)]^2}{N^2} \quad (11)$$

where $f(i, j)$ is the original image, and $\hat{f}(i, j)$ is the received image.

In the first experiment, Lenna image is transmitted using OFDM with and without chaotic interleaving with a bit energy to noise ratio $E_b/N_0 = 0$ dB over a single path Rayleigh fading channel in the presence of additive white Gaussian noise (AWGN). QPSK is used for baseband modulation. 128 subcarriers are used in OFDM and a guard interval length = 1/4 of the symbol duration is implemented. The results of this experiment are shown in Figure (5). It is clear from these results that the use of OFDM with chaotic interleaving increases the PSNR in the reconstructed image by about 6 dB.

Two other experiments are performed on the Lenna image but with $E_b/N_0 = 2$ and 4 dB, respectively. The results of these experiments are shown in Figures (6) and (7), respectively. These results reveal the superiority of the proposed OFDM transmission scheme with chaotic interleaving to that without chaotic interleaving.

Three other experiments are also performed on the Cameraman image and their results are tabulated in Table 1. These results also reveal the superiority of OFDM with chaotic interleaving to normal OFDM without interleaving.



(a) OFDM only.
PSNR= 27.8 dB.



(b) OFDM with
chaotic interleaving.
PSNR=33.8 dB.

Figure 5. The received Lenna image using OFDM with and without chaotic interleaving. $E_b/N_0 = 0$ dB and $f_d = 320$ Hz.



(a) OFDM only.
PSNR= 31.7 dB.



(b) OFDM with
chaotic interleaving.
PSNR=45 dB.

Figure 6. The received Lenna image using OFDM with and without chaotic interleaving. $E_b/N_0 = 2$ dB and $f_d = 320$ Hz.

E_b/N_0 (dB)	0	2	4
PSNR (dB) normal OFDM	27.9	33.4	35.7
PSNR (dB) OFDM with chaotic interleaving	33.3	45.2	66.2

Table 1. Results of the transmission of the Cameraman Image



(a) OFDM only.
PSNR= 35.9 dB.



(b) OFDM with
chaotic interleaving.
PSNR=100 dB.

Figure 7. The received Lenna image using OFDM with and without chaotic interleaving. $E_b/N_0 = 4$ dB and $f_d = 320$ Hz.

7. CONCLUSIONS

An efficient chaotic interleaving scheme has been presented for wireless image transmission with OFDM. This interleaving scheme is based on the chaotic Baker map. The interleaving process is applied to the binary image data prior to the modulation step. The proposed scheme improves the performance of the OFDM system, where it generates permuted sequences with lower correlation between their samples. The performance of the proposed scheme was studied over a Rayleigh multipath fading channel and the obtained results show a noticeable performance improvement over normal OFDM.

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