

Energy Detection Based Spectrum Sensing for Sensing Error Minimization in Cognitive Radio Networks

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Abstract: In this paper, we investigate an optimization of threshold level with energy detection to improve the spectrum sensing performance. Determining threshold level to minimize spectrum sensing error both reduces collision probability with primary user and enhances usage level of vacant spectrum, resulting in improving total spectrum efficiency. However, when determining threshold level, spectrum sensing constraint should also be satisfied since it guarantees minimum required protection level of primary user and usage level of vacant spectrum. To minimize spectrum sensing error for given spectrum sensing constraint, we derive an optimal adaptive threshold level by utilizing the spectrum sensing error function and constraint which is given by inequality condition. Simulation results show that the proposed scheme provides better spectrum sensing performance compared to conventional schemes.

Keywords: Cognitive radio, energy detection, spectrum sensing.

1. Introduction

Demand for ubiquitous wireless services requires the use of more spectrum resources. However, today's wireless networks are characterized by a fixed spectrum assignment policy. As a result, few spectrum resources such as 2.4 GHz unlicensed industrial, scientific, and medical (ISM) band are currently available for future wireless applications [1]. Operating in unlicensed band is risky since interference between heterogeneous systems degrades system performance.

To alleviate this problem, cognitive radio is being recognized as an intelligent technology due to its ability to rapidly and autonomously adapt operating parameters to changing environment [2], [3]. One important task for realizing cognitive radio is spectrum sensing since the devices need to reliably detect weak ongoing (or primary) signals [4]. In general, spectrum sensing techniques can be classified into three categories; energy detection, matched filter coherent detection, and cyclostationary feature detection [4]. Since non-coherent energy detection can be applied to anywhere and is able to locate spectrum occupancy information quickly, it is widely used in cognitive systems [5]–[10].

In spectrum sensing, it is desired to minimize spectrum sensing error (i.e., sum of false alarm and miss detection probabilities) since minimizing spectrum sensing error both reduces collision probability with primary user and enhances usage level of vacant spectrum. To provide reliable spectrum

sensing performance (i.e., minimize spectrum sensing error), one of the great challenges is determining threshold level since spectrum sensing performance depends on the threshold level. When determining threshold level, besides spectrum sensing error, spectrum sensing constraint which requires false alarm and miss detection probabilities to be below target level should also be considered since it guarantees minimum required protection level of primary user and usage level of vacant spectrum.

The optimal threshold level for minimizing spectrum sensing error (MSSE) was determined [5]. However, it does not consider spectrum sensing constraint, violating spectrum sensing constraint. To alleviate this problem, threshold level has been determined to provide constant detection rate (CDR) satisfying spectrum sensing constraint [8]–[10]. However, since the CDR only considers spectrum sensing constraint in determining threshold level, it cannot guarantee minimization of spectrum sensing error. In addition, the CDR can provide at most constant detection probability even in high SNR region where signal strength is much stronger than noise power to be easily distinguished.

In this paper, we consider an optimization of threshold level with energy detection to minimize the spectrum sensing error for a given sensing constraint. The false alarm and miss detection probabilities are monotonically increased and decreased, respectively, as the threshold level increases [4], [8]. Therefore, the spectrum sensing error function has concave or convex properties for certain threshold level duration. To optimize threshold level, besides spectrum sensing error, spectrum sensing constraint which is given by inequality condition should also be considered. Based on properties of spectrum sensing error function and inequality spectrum sensing constraint, we derive an adaptive optimal spectrum sensing threshold level minimizing spectrum sensing error while satisfying spectrum sensing constraint. Through the use of the proposed spectrum sensing scheme, the spectrum sensing performance can be improved compared to conventional schemes.

The rest of this paper is organized as follows. Section II describes the system model and Section III describes the proposed spectrum sensing scheme. Section IV verifies the performance of the proposed scheme by computer simulation. Finally, conclusions are given in Section V.

2. System Model

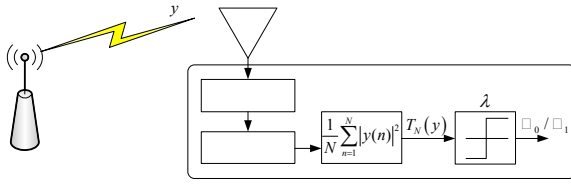


Fig. 1. System model for spectrum sensing

Fig. 1 depicts the system model for spectrum sensing, where cognitive (or secondary) user detects the presence of ongoing (or primary) user's signal using a hypothesis test. The received signal sample of a secondary user can be represented as

$$y(n) = \begin{cases} w(n); & \text{Ongoing user } H_0 \\ h(n)s(n) + w(n); & H_1 \end{cases} \quad (1)$$

where n denotes the sample index, $h(n)$ denotes the impulse response of the channel between the primary and secondary users, $s(n)$ is the signal from the primary user with zero mean and unit variance (i.e., $E\{|s(n)|^2\} = 1$), $w(n)$ denotes zero-mean circular-symmetric complex Gaussian (CSCG) noise with variance σ_w^2 (i.e., $w(n) \sim CN(0, \sigma_w^2)$), and H_0 and H_1 represent hypothesis corresponding to the absence and presence of the primary user's signal, respectively. For ease of analysis, we assume that the channel $h(n)$ is unchanged during the sensing process, say $h(n) = h_0$.

We consider the use of an energy detection for the spectrum sensing. Then, the test statistic for the energy detector can be represented as

$$T_N(y) = \frac{1}{N} \sum_{n=1}^N |y(n)|^2 \underset{H_1}{\overset{H_0}{\leq}} \lambda \quad (2)$$

where N is the number of samples and λ is the threshold level to be determined.

3. Proposed spectrum sensing scheme

We determine the threshold level for the energy detection to minimize the spectrum sensing error for a given spectrum sensing constraint. It can be shown that the test statistic $T_N(y)$ is a random variable having a chi-square distribution with $2N$ degrees of freedom. From the central limit theorem, $T_N(y)$ can be approximated as a Gaussian random variable with mean

$$\mu = \begin{cases} \sigma_u^2 \square \mu_0; & H_0 \\ \sigma_u^2 (\gamma + 1) \square \mu_1; & H_1 \end{cases} \quad (3)$$

and variance

$$\sigma^2 = \begin{cases} \frac{1}{N} \sigma_u^4 \square \sigma_0^2; & H_0 \\ \frac{\sigma_u^4}{N} (2\gamma + 1) \square \sigma_1^2; & H_1, \end{cases} \quad (4)$$

RF Antenna where $\gamma (= |h_0|^2 \sigma_s^2 / \sigma_u^2)$ is the received signal-to-noise power ratio (SNR).

Under hypothesis H_0 , the false alarm probability can be represented as

$$\begin{aligned} \text{Down Converter} \quad p_f(\lambda) &= \Pr(T_N(y) > \lambda | H_0) \\ \text{Sampling} \quad &= Q\left(\left(\frac{\lambda}{\sigma_u^2} - 1\right) \sqrt{N}\right) \end{aligned} \quad (5)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{z^2}{2}\right) dz \quad (6)$$

Similarly, under hypothesis H_1 , the detection probability can be represented as

$$\begin{aligned} p_d(\lambda) &= \Pr(T_N(y) > \lambda | H_1) \\ &= Q\left\{\left(\frac{\lambda}{\sigma_u^2} - \gamma - 1\right) \sqrt{\frac{N}{2\gamma + 1}}\right\} \end{aligned} \quad (7)$$

Thus, the miss detection probability can be represented as

$$p_m(\lambda) = 1 - p_d(\lambda) \quad (8)$$

From (5) and (7), for a target miss detection probability \bar{p}_m , the relation between false alarm and miss detection probabilities can be represented as

$$p_f(\lambda) = Q\left\{\sqrt{2\gamma + 1} Q^{-1}(1 - \bar{p}_m) - \gamma \sqrt{N}\right\} \quad (9)$$

Therefore, for a given pair of target probabilities (\bar{p}_f, \bar{p}_m) , the minimum number of required samples to achieve these targets can be determined by

$$N_{\min} = \left\lceil \frac{1}{\gamma^2} \left\{ Q^{-1}(\bar{p}_f) - Q^{-1}(1 - \bar{p}_m) \sqrt{2\gamma + 1} \right\}^2 \right\rceil \quad (10)$$

The lower the false alarm probability, the larger the capacity of the secondary user due to more chances to access to vacant spectrum. On the other hand, the lower the miss detection probability, the larger the capacity of the primary user due to high protection level about ongoing transmission. It can be seen from (5) and (7) that the spectrum sensing performance depends on threshold level. Therefore, it is

desired to determine the threshold level for the test statistic to minimize the spectrum sensing error (i.e., sum of false alarm and miss detection probability) while satisfying spectrum sensing constraints (i.e., $p_f(\lambda) \leq \bar{p}_f$ and $p_m(\lambda) \leq \bar{p}_m$) sufficiently.

The threshold level minimizing spectrum sensing error (MSSE) was determined [5], however, it does not consider the spectrum sensing constraint. As a result, the MSSE cannot guarantee the minimum protection level of primary user and usage level of vacant spectrum especially in low SNR region. Unlike the MSSE, sensing threshold was determined to provide constant detection rate (CDR) satisfying spectrum sensing constraint [8]–[10]. However, the CDR does not consider the minimization of spectrum sensing error and can provide at most constant detection probability even in high SNR region.

To alleviate above mentioned problems, we consider an optimization of threshold level to minimize the spectrum sensing error while satisfying spectrum sensing constraint sufficiently. Therefore, the level optimization problem can be represented as

$$\begin{aligned} & \min_{\lambda} F(\lambda) \\ & \text{s.t. } p_m(\lambda) \leq \bar{p}_m \end{aligned} \quad (11)$$

where $F(\lambda)$ is spectrum sensing error represented as

$$\begin{aligned} F(\lambda) &= p_f(\lambda) + p_m(\lambda) \\ &= Q\left(\frac{\lambda - \mu_0}{\sigma_0}\right) + \left\{1 - Q\left(\frac{\lambda - \mu_1}{\sigma_1}\right)\right\} \end{aligned} \quad (12)$$

Note that since we set the number of samples to achieve target pair of probabilities (\bar{p}_f, \bar{p}_m) as shown in (10), the threshold level satisfying (11) also satisfies false alarm constraint $p_f(\lambda) \leq \bar{p}_f$.

The spectrum sensing error function in (12) has global maximum and minimum values. Therefore, the threshold level minimizing spectrum sensing error can be achieved when $\frac{\partial F(\lambda)}{\partial \lambda} = 0$ and $\frac{\partial^2 F(\lambda)}{\partial^2 \lambda} > 0$. From (12), we obtain

$$\begin{aligned} \frac{\partial F(\lambda)}{\partial \lambda} &= \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left\{-\frac{(\lambda - \mu_0)^2}{2\sigma_0^2}\right\} \\ &+ \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left\{-\frac{(\lambda - \mu_1)^2}{2\sigma_1^2}\right\} \end{aligned} \quad (13)$$

and

$$\begin{aligned} \frac{\partial^2 F(\lambda)}{\partial^2 \lambda} &= \frac{(\lambda - \mu_0)}{\sqrt{2\pi}\sigma_0^3} \exp\left\{-\frac{(\lambda - \mu_0)^2}{2\sigma_0^2}\right\} \\ &+ \frac{(\lambda - \mu_1)}{\sqrt{2\pi}\sigma_1^3} \exp\left\{-\frac{(\lambda - \mu_1)^2}{2\sigma_1^2}\right\} \end{aligned} \quad (14)$$

The threshold level satisfying $\frac{\partial F(\lambda)}{\partial \lambda} = 0$ and $\frac{\partial^2 F(\lambda)}{\partial^2 \lambda} > 0$ can be derived from (13) and (14) as

$$\lambda' = \frac{-\beta + \sqrt{\beta^2 - \alpha\omega}}{\alpha} \quad (15)$$

where

$$\begin{aligned} \alpha &= \sigma_1^2 - \sigma_0^2 \\ \beta &= \sigma_0^2 \mu_1 - \sigma_1^2 \mu_0 \\ \omega &= \sigma_1^2 \mu_0^2 - \sigma_0^2 \mu_1^2 - 2\sigma_1^2 \sigma_0^2 \ln\left(\frac{\sigma_1}{\sigma_0}\right) \end{aligned} \quad (16)$$

In optimizing threshold level, we also consider the spectrum sensing constraint requiring to make the miss detection probability below maximum allowable miss detection probability (i.e., $p_m(\lambda) \leq \bar{p}_m$). Since $p_d(\lambda) = 1 - p_m(\lambda)$, the spectrum sensing constraint can be represented as

$$p_d(\lambda) = Q\left(\frac{\lambda - \mu_1}{\sigma_1}\right) \geq \bar{p}_d \quad (17)$$

where $\bar{p}_d (= 1 - \bar{p}_m)$ is minimum required detection probability. From (16), the threshold level providing minimum required detection performance can be represented as

$$\bar{\lambda} = Q^{-1}(\bar{p}_d) \sigma_1 + \mu_1 \quad (18)$$

Since $p_d(\lambda)$ is monotonically decreasing function of λ , if $\lambda' \leq \bar{\lambda}$, λ' is the optimal sensing threshold (λ^*) minimizing spectrum sensing error while satisfying spectrum sensing requirement sufficiently (i.e., $\lambda^* = \lambda'$ if $\lambda' \leq \bar{\lambda}$).

On the other hand, if $\lambda' > \bar{\lambda}$, λ' no more satisfies spectrum sensing constraint. In this case, the optimal threshold level λ^* should exist in following duration to satisfy spectrum sensing constraint.

$$0 \leq \lambda^* \leq \bar{\lambda}_d < \lambda' \quad (19)$$

From (13), the threshold levels satisfying $\frac{\partial F(\lambda)}{\partial \lambda} = 0$ can be represented as

$$\lambda_1 = \frac{-\beta + \sqrt{\beta^2 - \alpha\omega}}{\alpha} \quad (20)$$

and

$$\lambda_2 = \frac{-\beta - \sqrt{\beta^2 - \alpha\omega}}{\alpha} \quad (21)$$

Note that $\lambda_1 = \lambda'$. Since λ_1 is threshold level minimizing spectrum sensing error without considering spectrum sensing constraint and $\lambda_2 < 0$, it can be easily known that $F(\lambda)$ is monotonically decreased for $0 < \lambda < \lambda'$. Therefore, if $\lambda' > \bar{\lambda}$, $\bar{\lambda}$ becomes the optimal threshold λ^* minimizing spectrum sensing error while satisfying spectrum sensing constraint (i.e., $\lambda^* = \bar{\lambda}$ if $\lambda' > \bar{\lambda}$). Considering two cases of $\lambda' < \bar{\lambda}$ and $\lambda' > \bar{\lambda}$, the adaptive optimal threshold level minimizing spectrum sensing error while satisfying spectrum sensing constraint can be represented as

$$\lambda^* = \min\{\lambda', \bar{\lambda}\} \quad (22)$$

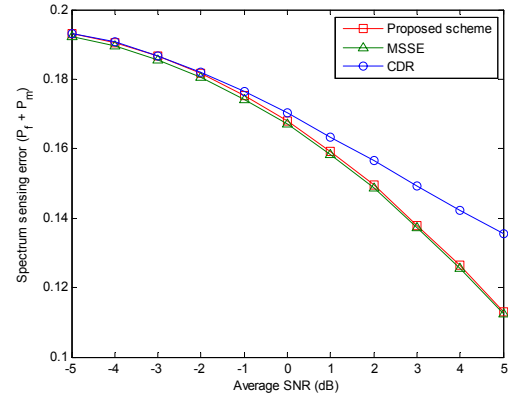
4. Simulation results

The performance of the proposed scheme is verified by computer simulation. We assume that the channel between secondary and primary user is Rayleigh faded. To verify the validation of the proposed scheme, we compare the performance of the proposed scheme with the MSSE and CDR spectrum sensing schemes.

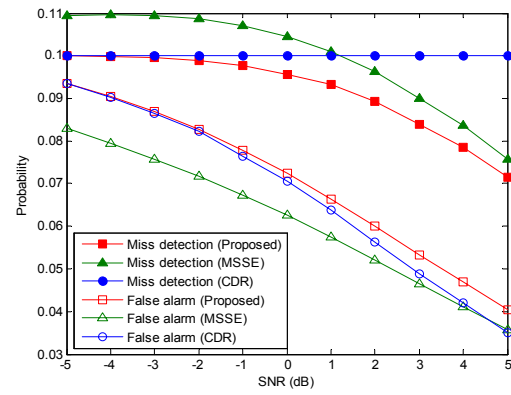
Fig. 2 depicts the local spectrum sensing performance with constraint $p_m(\lambda) \leq \bar{p}_m$ according to an average SNR when the maximum allowable miss detection probability $\bar{p}_m = 0.1$ (i.e., $\bar{p}_d = 0.9$). We set the number of samples for energy detection as (10) (i.e., $N = N_{\min}$). It can be seen that the spectrum sensing error is decreased as the average SNR increases regardless of spectrum sensing schemes. This is due to the fact that as the average SNR increases, interference signal power is much stronger than noise power, making it easy to distinguish between present and absent of primary user. It can also be seen that the proposed spectrum sensing scheme minimizes spectrum sensing error while satisfying spectrum sensing constraint sufficiently. Although the MSSE provides best spectrum sensing error performance, it violates spectrum sensing constraint as shown in Fig. 2 (b), thus is inadequate to perform spectrum sensing.

Fig. 3 depicts the spectrum sensing performance with constraint $p_m(\lambda) \leq 0.1$ according to the number of samples when the average SNR is -3 and 3 dB. It can be seen that for a given number of samples, the proposed scheme provides better spectrum sensing performance than the CDR. This is due to the fact that unlike the CDR determining threshold level to meet $p_m(\lambda) = \bar{p}_m$, the proposed scheme adaptively

optimizes threshold level according to spectrum sensing environment. It can also be seen that the proposed scheme provides spectrum sensing performance similar to the MSSE while satisfying spectrum sensing constraint as the number of samples increase.



(a) Spectrum sensing error



(b) False alarm and miss detection probabilities

Fig. 2. Spectrum sensing performance according to an average SNR

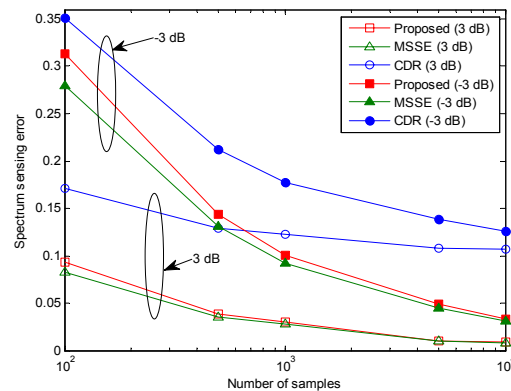


Fig. 3. Spectrum sensing performance according to the number of samples

5. Conclusions

In this paper, we considered the optimization of threshold level with energy detection to minimize the spectrum sensing error for a given inequality spectrum sensing constraint. By considering both property of spectrum sensing error function and inequality spectrum sensing constraint, we derived optimal adaptive threshold level. Through the use of the proposed sensing threshold, spectrum sensing error can be minimized while satisfying spectrum sensing requirement.

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